At this point, use the following vector identity:

$$A \times (B \times C) \equiv (A \cdot C)B - (A \cdot B)C \tag{24}$$

to show that

$$-\Omega \times (\rho \times \Delta R) = -(\Omega \cdot \Delta R)\rho + (\Omega \cdot \rho)\Delta R \tag{25}$$

$$-(\rho \times \Omega) \times \Delta R = (\Delta R \cdot \Omega)\rho - (\Delta R \cdot \rho)\Omega \tag{26}$$

and

$$\rho \times (\Omega \times \Delta R) = (\rho \cdot \Delta R)\Omega - (\rho \cdot \Omega)\Delta R \tag{27}$$

The sum of the right-hand side of Eqs. (25-27) vanishes; thus, summation of these equations yields

$$-\Omega \times (\rho \times \Delta R) - (\rho \times \Omega) \times \Delta R + \rho \times (\Omega \times \Delta R) = 0 \tag{28}$$

Therefore, the terms containing ΔR can be eliminated from Eq. (23) which results in

$$\Delta V_t + (2\Omega + \rho) \times \Delta V = J \tag{29}$$

After the substitution of the expression for J given in Eq. (7) into Eq. (29) the latter becomes

$$\Delta V_t + (2\Omega + \rho) \times \Delta V = \nabla - \psi \times f + \Delta g + \delta g \tag{30}$$

This ends the proof since Eqs. (30) and (2) are identical.

Conclusion

The position and velocity error models given in Eqs. (1) and (2), respectively, are derived in the literature through perturbation of the respective nominal equations [Eqs. (3) and (6), respectively]. The two nominal equations stem from the same physical origin expressed in Eq. (4). Although Eqs. (1) and (2) are assumed identical, to the best of our knowledge a direct proof of this proposition was never furnished in the literature. This Note completes the triangle by providing such proof. The proof is accomplished by operations on Eq. (1) which eventually yield Eq. (2). The operations involve kinematical theorems as well as vector algebra.

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Navigation Accuracy Analysis for an Ion Drive Rendezvous with Comet Tempel 2

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Introduction

REFERENCES 1 and 2 have discussed various aspects of the navigation of a proposed dual comet mission using the Solar Electric Propulsion System (SEPS). Reference 1 investigates the early portion of this mission, a fast flyby of Comet Halley, including delivery of an atmospheric probe. Reference 2 investigates later parts of the mission, including a powered heliocentric cruise phase, an approach to rendezvous with Comet Tempel 2, and subsequent operation near the cometary nucleus. Information regarding the scientific objectives, mission design, and spacecraft design for this mission is presented in Refs. 1-3. Included in Ref. 2 are estimated orbit determination and guidance accuracies for the Tempel 2 rendezvous approach phase of the mission, including a number of sensitivity studies involving parameters such as data frequencies, data accuracies, ion drive thrust vector errors, comet ephemeris uncertainties, and time lags associated with data processing and command sequence generation. The rendezvous accuracies stated in Ref. 2 are substantially better than those stated in Ref. 4 for another ion drive comet rendezvous mission. In this paper, the approach to rendezvous with Tempel 2 is re-examined. Rendezvous accuracies significantly better than those presented in Ref. 2 are obtained. The parametric sensitivity results of Ref. 2 are updated. In addition, the sensitivity of the rendezvous accuracy to the limits placed on thrust vector changes for guidance purposes is investigated, along with the sensitivity to guidance mode (fixed vs variable time of arrival). Although this particular comet rendezvous mission is no longer under consideration by NASA for a new program start, due to funding limitations, the mission is still of interest, since it is representative of SEP comet rendezvous missions in general.

Navigation Strategy

The navigation system for this mission consists of the Deep Space Network (DSN), which provides radiometric tracking data, elements of the flight system (imaging science subsystem, ion drive thrusters, and perhaps a radar altimeter or three-axis accelerometer), ground-based astronomical observatories, and ground-based computational facilities and software.[1,2] The Tempel 2 rendezvous approach phase begins 60 days before initial rendezvous, at which time the comet should be detectable with the narrow angle imaging system on board the spacecraft. Navigation tracking cycles[1] will be scheduled every five days throughout the approach phase. A single-station pass of two-way coherent doppler data, with ranging, will be needed each day, outside of the navigation tracking cycles. Ground-based comet observations will be

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Table 1 Position uncertainty standard deviations and position standard deviations mapped to rendezvous

	Mapped position uncertainty, km			Mapped position, km		
Time	Downtrack	Crosstrack	Out-of-plane	Downtrack	Crosstrack	Out-of-plane
R − 58	18,221	17,786	17,877	18,221	17,786	17,877
R − 48	10,455	10,487	10,568	20,849	29,220	27,930
R − 38	6591	6642	6696	15,485	18,819	16,220
R − 28	3732	3568	3584	10,660	9330	8361
R − 18	1928	1494	1499	3665	2881	2783
R − 8	958	342	325	1540	688	680
R − 4	212	163	131	679	237	200
R	16	9	7	373	85	67

made every five days. For the first 20 days of this mission phase, optical navigation imaging frames (of the comet and background catalogued stars) will be recorded twice a day. For the remaining 40 days they will be recorded four times a day. Approach phase navigation relies heavily on onboard optical data because of the cometary ephemeris uncertainty. The accuracy of the orbit determination process is dependent upon the accuracy with which nongravitational accelerations acting on the flight system (due to thrust and solar radiation pressure, in this mission phase) are known. The desired uncertainties in knowledge and control of these accelerations are substantially the same as in the Halley flyby phase of the mission.[1]

During the approach to rendezvous, estimated trajectory dispersions are corrected by suitably modifying the thrust vector history up to the time of initial rendezvous (and perhaps shifting the time of rendezvous also). In the more ambitious version of this mission,[1] it is desired to achieve rendezvous with Tempel 2 at least 60 days prior to perihelion. In the less ambitious version, rendezvous is desired at least 30 days prior to perihelion. In either case, the desired initial rendezvous point is several thousand kilometers from the nucleus, safely outside the dust envelope. After the first several navigation imaging frames have been acquired and an orbit determination solution obtained, the trajectory will be retargeted (i.e., a new nominal trajectory and a corresponding thrust vector history searched out). Subsequent to this retargeting, the thrust vector commands will be periodically updated using a linear guidance scheme. The control gains for the linear guidance will be computed only once, and then stored, thus significantly reducing the computational effort during the high-activity approach period and shortening the time needed for control updates. Thrust vector commands will be updated every five days, until eight days before rendezvous. During the last eight days, thrust vector commands will be updated every two days.

Navigation Accuracy Analysis and Parametric Sensitivity Studies

Covariance analyses have been carried out to investigate the statistical properties of these orbit determination and guidance strategies. The parameters which were estimated in carrying out the orbit determination solution were the flight system position, velocity, and mass, the cometary ephemeris, and thrust vector errors. The solution was obtained using a sequential filtering procedure. The thrust vector errors were represented as independent variations in three orthogonal directions. The error in each direction was modeled as the sum of two independent Gauss-Markov processes, characterized by different correlation times, so as to represent both short-term and long-term effects. Parameters which were considered, rather than estimated, in carrying out the orbit determination solution include equivalent station location errors[1] for stations 14, 43, and 63 (errors in station spin radius and longitude were included), differenced range biases between stations 14 and 63 and stations 43 and 63, and differenced frequency biases between the same station pairs. Also included as considered parameters were biases in the onboard optical data. A priori standard deviations for the various parameters and correlation times for the random processes were the same as in Ref. 1, for the Halley flyby mission phase, except that the cometary position components were assumed known to 3000 km (1σ), rather than 5000 km. Noise levels for the various data types were also the same as in Ref. 1. A batch size of twelve hours was used in carrying out the orbit determination solution.

Corrections in the thrust magnitude and direction were computed every five days, based upon estimated perturbations in position, velocity, bias parameters, and stochastic parameters, between $R-58$ and $R-8$ (R = rendezvous, numbers denote days relative to rendezvous). These control corrections were chosen to be piecewise constant over 2½ day intervals. Control corrections were computed every two days between $R-8$ and R, and were chosen to be piecewise constant over one-day intervals. Standard deviations of control corrections were limited to 5% in thrust magnitude and 0.1 rad in the two thrust angles. Fixed time of arrival guidance was used, but a final time correction for improved rendezvous was computed also. The minimum miss terminal guidance technique described in Ref. 5 was the perturbation guidance technique employed.

In Table 1, standard deviations of the spacecraft position uncertainty and the spacecraft position, both mapped deterministically from the current time to the time of rendezvous, are presented. Note that the spacecraft state is referenced to that of the comet, not that of the sun or Earth. The spacecraft state uncertainties are those errors associated with the spacecraft state estimate at time t, based upon data through time $t-1$, one day being the assumed data turnaround time for orbit determination and guidance purposes. The spacecraft state errors represent trajectory dispersions, i.e., deviations of the actual trajectory from the nominal trajectory. The "downtrack" direction is along the spacecraft flight path, in a comet-centered coordinate frame; the "crosstrack" direction is perpendicular to the flight path and in the spacecraft comet-relative orbit plane; and the "out-of-plane" direction completes an orthogonal triad.

Although the guidance strategy is designed to achieve rendezvous at a fixed time, the downtrack velocity error at rendezvous can be reduced significantly by shifting the time at which rendezvous is defined to occur. The required time shift is 13.8 min (1σ). With this adjustment in time taken into account, downtrack, crosstrack, and out-of-plane position error standard deviations at rendezvous are 373, 85, and 67 km, respectively. The corresponding velocity error standard deviations at rendezvous are 0.109, 0.255, and 0.232 m/s. Without this time of flight correction, the downtrack velocity error standard deviation is 0.234 m/s at the nominal rendezvous time. (The other five error standard deviations change very little.) The downtrack position errors are larger than the crosstrack and out-of-plane position errors due to the comet ephemeris uncertainty, and the fact that the optical data provide relatively little information in the downtrack direction until near encounter. The rms position error at rendezvous is about one-third the value reported in Ref. 2.

Table 2 Position and velocity standard deviations at rendezvous: parametric sensitivity study

Case	Position standard deviations, km			Velocity standard deviations, m/s		
	Downtrack	Crosstrack	Out-of-plane	Downtrack	Crosstrack	Out-of-plane
BL	373	85	67	.109	.255	.232
VTA	98	65	55	.109	.255	.232
DT*2	206	90	71	.109	.255	.232
IL*2	539	130	103	.155	.360	.321
OF/2	425	100	79	.113	.294	.264
GN*2	619	92	71	.111	.255	.232
ACE*2	456	87	69	.117	.255	.232
TF/2	491	83	67	.125	.255	.232
NDD	499	89	70	.152	.254	.232
CTR	232	76	62	.093	.255	.233
TAE*2	373	85	67	.124	.258	.235

The rms position and velocity errors at rendezvous in Ref. 2 are, in turn, less than one-third the values presented in Ref. 4 for a Halley rendezvous mission. The reasons for a difference between the results here and those in Ref. 2 are as follows: 1) the thrust vector error model has been revised, with new standard deviations and correlation times; 2) data noise assumptions for differenced range and ground-based optical data have been changed; 3) six, rather than four, of the estimated stochastic parameters have been made available for feedback in the guidance law; 4) the orbit determination data batch size has been increased; 5) onboard optical data biases have been included as considered parameters; 6) the upper limit on thrust magnitude changes for guidance purposes has been reduced; and 7) a correction has been made in rendezvous targeting. Items 1-3 tend to improve the calculated rendezvous accuracy; items 5 and 6 tend to degrade it; item 7 was verified to have little effect; and item 4, resulting in reduced computational cost and allowed as a consequence of item 1, should have little effect on rendezvous accuracy.

In addition to the accuracy analysis described above for the baseline navigation strategy, studies were carried out to determine the sensitivities of the navigation accuracies to changes in the navigation strategy and various error modeling assumptions. In each instance, all assumptions were exactly as in the baseline case, with one assumption changed. In Table 2, position and velocity standard deviations at rendezvous are presented for ten such cases, plus the baseline case (BL).

Rendezvous accuracies were found to improve substantially if variable, rather than fixed, time-of-arrival guidance were used (case VTA). In this situation, one has an additional control parameter, time-of-arrival, to make use of in formulating control logic to drive expected position and velocity errors at rendezvous to zero. (The variable time-of-arrival guidance under discussion here is not to be confused with the shift in arrival time mentioned previously in conjunction with a fixed time-of-arrival guidance scheme. In the latter, all thrust vector feedback gain calculations assume arrival at a fixed time. The time of thruster shutoff is then updated at the last minute, based upon the most recent orbit determination solution, to reduce to zero the estimated velocity along the nominal flight path.) By allowing a variation in time of rendezvous of 5.8 h (1σ), the position error at rendezvous can be reduced by roughly a factor of three. Since the exact time of initial rendezvous is not of critical importance, this would seem to be an advantageous trade. (Science requirements dealing with time-of-arrival are expressed in terms of days from perihelion rather than hours; moreover, there are no apparent operational requirements for thruster shutoff at a time established months in advance.) An increase in the allowed variation in thrust magnitude for guidance purposes [from 5% of nominal to 10% (1σ)], improves the downtrack position error by 40% (case DT*2). However, such a change also requires a doubling of the power reserve from 10 or 15% to 20 or 30%, if thrust acceleration levels 2 or 3 standard deviations above the mean are to be available when needed. An increase in power reserve implies either a SEPS modification to produce additional power or a more gradual approach to rendezvous. The latter situation implies either a later rendezvous or a smaller delivered payload, due to a less efficient heliocentric trajectory. A doubling (to 48 h), of the information lag associated with the processing of data to be used for orbit determination purposes, generation of an orbit determination solution, computation of thrust vector parameters, development of command sequences, and round-trip light time, produces about a 50% degradation in rendezvous accuracy (case IL*2).

The remaining sensitivity studies deal with parameters which affect the accuracy of the orbit determination process. Halving the frequency of onboard optical data (case OF/2) causes a noticeable degradation in the rendezvous accuracy. A doubling of the noise in the ground-based optical data (case GN*2) produced a substantial worsening of the downtrack position standard deviation at rendezvous. The crosstrack and out-of-plane position standard deviations were affected less, because the onboard optical data provide the most important information in these directions. A doubling of the a priori cometary ephemeris uncertainty (both the position and velocity components—case ACE*2) produced changes in rendezvous accuracy which are qualitatively similar to those for case GN*2.

Three sensitivity studies involving radiometric tracking data were carried out. The first involved a reduction in the frequency of navigation tracking cycles from one every five days to one every ten days (case TF/2). The data gaps created were filled with a single-station pass of conventional doppler and range data per day. The downtrack position and velocity standard deviations increased somewhat. There was little change in the other two directions, due to a heavy dependence of these results on onboard optical data. The complete elimination of differenced multistation radiometric data (case NDD) produced results which were qualitatively similar to those for case TF/2, but somewhat worse. To further examine the importance of radiometric tracking data, a scenario was considered in which doppler and range data were available 24 hours a day, with differenced multistation data available during all station overlaps (case CTR). This situation is essentially a continuous pattern of navigation tracking cycles. It is unrealistic because of manpower costs and commitments of the DSN to other flight projects, but provides a lower bound on navigation accuracies achievable with a large quantity of radiometric data, in addition to the baseline amounts of onboard and ground-based optical data. Modest improvements were obtained in crosstrack and out-of-plane position standard deviations at rendezvous. More substantial improvements were obtained in downtrack position and velocity standard deviations.

The final sensitivity study involved increasing thrust acceleration errors above the levels assumed in the orbit determination filter error model (case TAE*2). With these errors doubled (in all three directions), there was no degradation in rendezvous position accuracy and a modest degradation in rendezvous velocity accuracy.

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Technical Comments

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Comment on "Stability of a Precision Attitude Determination Scheme"

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In Ref. 1, the stability of an attitude determination scheme discussed in Ref. 2 is investigated. The purposes of this Comment are 1) to show that certain conclusions reached in Ref. 1 are incorrect, and 2) to show that other conclusions, while correct, can be deduced more simply.

By way of introduction, consider a spacecraft which is nominally fixed in orientation relative to inertial space. Let ψ denote the rotation angle of the spacecraft about an inertially fixed axis and let ω denote the angular rate of the spacecraft about a nominally coincident body-fixed axis. Then

$$\dot{\psi} \approx \omega \tag{1}$$

Assume that the gyroscopically measured angular rate about the body-fixed axis can be modeled as

$$\omega_g = \omega - d - \eta_v \tag{2}$$

where

$$\dot{d} = \eta_u \tag{3}$$

and η_v and η_u are statistically stationary zero-mean white noise processes. The spacecraft dynamics about this body-fixed axis may be modeled as

$$I\dot{\omega} \approx T_c + T_d \tag{4}$$

where I, T_c, and T_d denote the spacecraft moment of inertia, the control torque, and the disturbance torque about this axis, respectively. If the control torque is chosen to be

$$T_c = -k(\omega_g - u) \tag{5}$$

where u is the commanded angular rate, then by choosing k sufficiently large, the difference $\omega_g - u$ could be made arbitrarily small, were it not for the white noise term in Eq. (2). This can be seen by substituting Eqs. (2) and (5) into Eq. (4). The approximation

$$\omega_g \approx u \tag{6}$$

is often made despite the white noise term in Eq. (2), yielding

$$\dot{\psi} \approx d + u + \eta_v \tag{7}$$

Equations (3) and (7) are equivalent to Eq. (1) in Ref. 1 and Eqs. (1) and (3) in Ref. 2. The purpose of this derivation has been to establish the assumptions and approximations typically associated with these equations.

It is argued in Ref. 1 that the fourth-order system consisting of Eqs. (3) and (7) and analogous equations for propagating optimal estimates of ψ and d (based upon discrete noisy measurements of ψ) is not stable. This conclusion is correct, but the sequence of eighteen equations used to reach this conclusion is not necessary. It follows from Eq. (3) that the variance of d increases linearly with time. Thus, if system instability in a stochastic problem is interpreted to mean that certain covariance matrix elements grow without bound as time becomes large, any system of equations including Eq. (3) is unstable.

In addition, certain arguments used in the derivation of this result in Ref. 1 are not correct. In particular, if A, B, and H denote system dynamics, control distribution, and measurement distribution matrices for a time-invariant linear system, controllability of the pair (A,B) is not a necessary condition for existence of a stable solution to the infinite time regulator problem. Nor is observability of the pair (A,H) necessary for existence of a stable solution to the infinite time estimator problem. Both these conditions are elements of sets of sufficient conditions for stability,[3] but are not necessary. If, for example, the term $-d/\tau$ $(\tau > 0)$ were added to the right-hand side of Eq. (3), the system of Eqs. (3) and (7) (ignoring the white noise terms) would remain uncontrollable, but could be stabilized by a suitable choice of u.

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